Improvement of the Finite Element Analysis of 3D, Nonlinear, Periodic Eddy Current Problems Involving Voltage Driven Coils under DC Bias

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Different techniques are investigated to improve the numerical solution process by the means of the finite element method of 3-dimensional, time-periodic, nonlinear eddy current problems under the influence of DC bias applying voltage excitation. The procedures of treating the equation system including DC bias are applied to 3-dimensional models of one- and three-phase benchmark transformer problems. The underlying nonlinear iteration technique is the time-periodic fixed-point method. Potential benefits of a static initialization procedure are investigated.

***Index Terms*—Eddy currents, nonlinear magnetics, finite element analysis, computational electromagnetics, geomagnetism.**

I. Introduction

The main goal of this investigation is to analyze different ways to formulate the equation system for solving 3-dimensional nonlinear, time-periodic finite element problems when a direct current (DC) bias is present in the excitation. Direct currents can occur e.g. due to geo-magnetically induced currents (GIC) [1], [2]. These DC components can cause adverse problems in power distribution infrastructures as e.g. power transformers due to adding a DC bias to the magnetization current of the transformer. As a consequence the core of the transformer gets saturated within the half-period in which the magnetization current and the DC bias are in the same direction resulting in increasing noise level, additional core losses as well as eddy current losses due to higher leakage flux [3], [4]. To predict these effects, numerical investigations of such waveforms have to be done. Hence a simplified benchmark problem of a single phase and a three-phase transformer are investigated by the means of a 3D finite element analysis.

The transformer model is assumed to be voltage driven. The focus of this work is to investigate the treatment of the DC components in the equation system obtained by the time-periodic fixed-point technique (TPFP) using the $\boldsymbol{T},\phi$-ϕ formulation [5], [6]. The fixed-point technique can be improved by choosing a good initial solution leading to a faster convergence to reduce the number of non-linear iterations. In case of a transformer problem, the eddy-current domains are relatively small compared to the non-conducting domains hence it is obvious to choose the initial solution by neglecting the eddy currents. This way to determine the initial values for the nonlinear iteration process will be called static initialization.

II. FEM Formulation

The numerical problem is solved by the use of the finite element method in terms of a current vector potential $\boldsymbol{T}$ and a magnetic scalar potential ϕ known as $\boldsymbol{T},\phi$-ϕ formulation as described in [4]-[7]. The ordinary differential equation system obtained in [4] can be written in a more compact form as:

$$\begin{bmatrix}\boldsymbol{R} & 0\\ 0 & \boldsymbol{S}_\rho\end{bmatrix}\begin{bmatrix}\boldsymbol{i}\\ \boldsymbol{x}\end{bmatrix}+{d}/{dt}\left(\begin{bmatrix}\boldsymbol{V}_\mu & \boldsymbol{g}_\mu\\ \boldsymbol{g}_\mu^T & \boldsymbol{C}_\mu\end{bmatrix}\begin{bmatrix}\boldsymbol{i}\\ \boldsymbol{x}\end{bmatrix}\right)=\begin{bmatrix}\boldsymbol{u}\\ 0\end{bmatrix}-\begin{bmatrix}{\mathrm{d}}/{\mathrm{d}t}\left(\boldsymbol{g}_\mu\boldsymbol{T}_0\right)\\ {\mathrm{d}}/{\mathrm{d}t}\left(\boldsymbol{C}_\mu\boldsymbol{T}_0\right)\end{bmatrix} \tag{1}$$

where $\boldsymbol{R}$ contains the ohmic resistances and the matrix $\boldsymbol{S}_\rho$ depends on the resistivity ρ. $\boldsymbol{C}_\mu$ results from the FEM basis functions. The matrices $\boldsymbol{V}_\mu$, $\boldsymbol{g}_\mu$, correspond to the impressed current vector potentials due to unit currents in the windings [8]. We have $\boldsymbol{i}=\left[i_1\cdots i_{n_C}\right]^\mathrm{T}$ as the vector of unknown currents and $\boldsymbol{u}=\left[u_1\cdots u_{n_C}\right]^\mathrm{T}$ is formed by the given voltages, where n_C is the number of voltage driven coils. The vector $\boldsymbol{x}$ gathers the unknown potentials $\boldsymbol{T}$ and ϕ.

In a DC bias problem, an additional condition has to be satisfied for the currents in (1):

$$\frac{1}{T}\int_0^T \boldsymbol{i}\,\mathrm{d}t=\boldsymbol{i}_{DC} \tag{2}$$

where $\boldsymbol{i}_{DC}$ represents the known direct currents in the windings and T is the length of one time-period.

III. Static Initialization

Performing a static initialization with the resistances of the coils also neglected, (1) can be simplified to:

$$\mathrm{d}/\mathrm{d}t\left(\begin{bmatrix}\boldsymbol{V}_\mu & \boldsymbol{g}_\mu\\ \boldsymbol{g}_\mu^T & \boldsymbol{C}_\mu\end{bmatrix}\begin{bmatrix}\boldsymbol{i}\\ \boldsymbol{x}\end{bmatrix}\right)=\begin{bmatrix}\boldsymbol{u}\\ \boldsymbol{0}\end{bmatrix}-\begin{bmatrix}{\mathrm{d}}/{\mathrm{d}t}\left(\boldsymbol{g}_\mu\boldsymbol{T}_0\right)\\ {\mathrm{d}}/{\mathrm{d}t}\left(\boldsymbol{C}_\mu\boldsymbol{T}_0\right)\end{bmatrix} \tag{3}$$

Due to the fact that $\boldsymbol{T}$ = 0, the number of degrees of freedom is essentially reduced in (3). The voltages can be computed as the time derivatives of the magnetic fluxes $\boldsymbol{\Phi}$ as

$$\boldsymbol{u}=-\frac{\mathrm{d}\boldsymbol{\Phi}}{\mathrm{d}t} \tag{4}$$

Integrating (3) over time results in:

$$\begin{bmatrix}\boldsymbol{V}_\mu & \boldsymbol{g}_\mu\\ \boldsymbol{g}_\mu^T & \boldsymbol{C}_\mu\end{bmatrix}\begin{bmatrix}\boldsymbol{i}\\ \boldsymbol{x}\end{bmatrix}=-\begin{bmatrix}\boldsymbol{\Phi}+\boldsymbol{\Phi}_0\\ \boldsymbol{0}\end{bmatrix}-\begin{bmatrix}\boldsymbol{g}_\mu\boldsymbol{T}_0\\ \boldsymbol{C}_\mu\boldsymbol{T}_0\end{bmatrix} \tag{5}$$

where $\boldsymbol{\Phi}_0$ is a vector of time independent constants built of the direct components of the magnetic fluxes. These are unknown and have to be determined to satisfy (2). Two different approaches to do this will be investigated.

A. Secant Method

As a first approach the secant-method [9] is used to iterate the direct component of the flux. This method has already been used in our previous work [4]. Let us assume that the DC flux Φ_{0j} of the *j*-th voltage driven coil depends on its DC current $i_{\mathrm{DC}j}$ only. Hence, the functions

$$f\left(\Phi_{0j}\right)=i_{0j} \tag{6}$$

can be defined as obtained from the solution of (5) for each discretized time step with a fixed Φ_{0j} and

$$i_{0j}=\frac{1}{n}\sum_{k=1}^{n} i_j\left(t_k\right) \tag{7}$$

where *n* is the number of time steps within one period. The secant-method can be applied to solve the equation $f\left(\Phi_{0j}\right)-i_{DCj}=0$ as:

$$\Phi_{0j}^{(k+1)}=\Phi_{0j}^{(k)}-\left(f\left(\Phi_{0j}^{(k)}\right)-i_{DCj}\right)\frac{\Phi_{0j}^{(k)}-\Phi_{0j}^{(k-1)}}{\left(i_{0j}^{(k)}-i_{0j}^{(k-1)}\right)} \tag{8}$$

with the initial conditionj $\Phi_{0j}^{(0)}=0$. This approach is a fast second order method, but numerical experiments show that its convergence rate is sensitive to the accuracy in evaluating the function *f* depending on the precision of the nonlinear material values in each time-step.

B. Schur Complement

In this approach the exciting flux of each coil will be calculated from the given exciting voltage of the coil for each nonlinear iteration step as

$$\Phi_j(t)=-\int_0^t u_j(t')\,\mathrm{d}t' \;. \tag{9}$$

The corresponding static equation system in the *n*-th time step has been written in (5).Equation (2) will be added to this system to ensure obtaining the applied direct current. The DC flux components $\mathbf{\Phi}_0$ have to be determined. A symmetric matrix system of the form

$$\begin{bmatrix} \begin{bmatrix} \boldsymbol{V}_{\mu,1} & \boldsymbol{g}_{\mu,1} \\ \boldsymbol{g}_{\mu,1}^T & \boldsymbol{C}_{\mu,1} \end{bmatrix} & & & \begin{bmatrix} -\boldsymbol{I} \\ 0 \end{bmatrix} \\ & \ddots & & \vdots \\ & & \begin{bmatrix} \boldsymbol{V}_{\mu,n} & \boldsymbol{g}_{\mu,n} \\ \boldsymbol{g}_{\mu,n}^T & \boldsymbol{C}_{\mu,n} \end{bmatrix} & \begin{bmatrix} -\boldsymbol{I} \\ \boldsymbol{0} \end{bmatrix} \\ \begin{bmatrix} -\boldsymbol{I} & \boldsymbol{0} \end{bmatrix} & \cdots & \begin{bmatrix} -\boldsymbol{I} & \boldsymbol{0} \end{bmatrix} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \boldsymbol{i}_1 \\ \boldsymbol{x}_1 \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \boldsymbol{i}_n \\ \boldsymbol{x}_n \end{bmatrix} \\ \Phi_0 \end{bmatrix} = -\begin{bmatrix} \begin{bmatrix} \Phi_1 \\ \boldsymbol{0} \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \Phi_n \\ \boldsymbol{0} \end{bmatrix} \\ n\boldsymbol{i}_{DC} \end{bmatrix} - \begin{bmatrix} \begin{bmatrix} \boldsymbol{g}_{\mu,1}\boldsymbol{T}_{0,1} \\ \boldsymbol{C}_{\mu,1}\boldsymbol{T}_{0,1} \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \boldsymbol{g}_{\mu,n}\boldsymbol{T}_{0,n} \\ \boldsymbol{C}_{\mu,n}\boldsymbol{T}_{0,n} \end{bmatrix} \\ \boldsymbol{0} \end{bmatrix}, \tag{10}$$

is obtained where $\boldsymbol{I}$ is an identity matrix with the size n_C. Applying the Schur complement [10] to the DC component of the flux, the equation system to be solved becomes

$$\begin{gathered} \left(-\sum_{k=1}^{n}\begin{bmatrix} -\boldsymbol{I} & 0 \end{bmatrix}\begin{bmatrix} \boldsymbol{V}_{\mu,k} & \boldsymbol{g}_{\mu,k} \\ \boldsymbol{g}_{\mu,k}^T & \boldsymbol{C}_{\mu,k} \end{bmatrix}^{-1}\begin{bmatrix} -\boldsymbol{I} \\ \boldsymbol{0} \end{bmatrix}\right)\mathbf{\Phi}_0 = \\ -n\boldsymbol{i}_{DC}-\sum_{k=1}^{n}\begin{bmatrix} -\boldsymbol{I} & \boldsymbol{0} \end{bmatrix}\begin{bmatrix} \boldsymbol{V}_{\mu,k} & \boldsymbol{g}_{\mu,k} \\ \boldsymbol{g}_{\mu,k}^T & \boldsymbol{C}_{\mu,k} \end{bmatrix}^{-1}\begin{bmatrix} -\mathbf{\Phi}_k-\boldsymbol{g}_{\mu,k}\boldsymbol{T}_{0,k} \\ -\boldsymbol{C}_{\mu,k}\boldsymbol{T}_{0,k} \end{bmatrix} \end{gathered} \;. \tag{11}$$

To solve (11), one can split the system and compute the components for each time step in parallel. Indeed, using the notations

$$\boldsymbol{D}_k := \begin{bmatrix} \boldsymbol{V}_{\mu,k} & \boldsymbol{g}_{\mu,k} \\ \boldsymbol{g}_{\mu,k}^T & \boldsymbol{C}_{\mu,k} \end{bmatrix}^{-1}\begin{bmatrix} -\boldsymbol{I} \\ \boldsymbol{0} \end{bmatrix},\; \boldsymbol{E}_k := \begin{bmatrix} \boldsymbol{V}_{\mu,k} & \boldsymbol{g}_{\mu,k} \\ \boldsymbol{g}_{\mu,k}^T & \boldsymbol{C}_{\mu,k} \end{bmatrix}^{-1}\begin{bmatrix} -\mathbf{\Phi}_k-\boldsymbol{g}_{\mu,k}\boldsymbol{T}_{0,k} \\ -\boldsymbol{C}_{\mu,k}\boldsymbol{T}_{0,k} \end{bmatrix} \tag{12}$$

where *k* represents a time-step between 1 and *n*, the Schur complement solution can be determined as:

$$\boldsymbol{S}=\sum_{k=1}^{n}\begin{bmatrix} -\boldsymbol{I} & \boldsymbol{0} \end{bmatrix}\boldsymbol{D}_k \;\text{ and }\; \boldsymbol{r}=-n\boldsymbol{i}_{DC}-\sum_{k=1}^{n}\begin{bmatrix} -\boldsymbol{I} & \boldsymbol{0} \end{bmatrix}\boldsymbol{E}_k \;. \tag{13}$$

Once all time steps have been solved, one can compute the DC component of the magnetic flux as

$$\mathbf{\Phi}_0=-\boldsymbol{S}^{-1}\boldsymbol{r} \;, \tag{14}$$

and, finally, to get the unknown vector $\boldsymbol{x}$ and the current for the static initialization, one has to compute for all time steps:

$$\begin{bmatrix} \boldsymbol{x}_k \\ \boldsymbol{i}_k \end{bmatrix}=\boldsymbol{E}_k+\boldsymbol{D}_k\mathbf{\Phi}_0 \,. \tag{15}$$

This procedure is repeated until the nonlinear material properties converge. The resulting unknown vector $\boldsymbol{x}$ will be the initial solution for starting the non-linear iteration process including eddy currents.

This method is independent of the accuracy of the evaluation of the non-linear material values, due to the exact determination of $\mathbf{\Phi}_0$ in each non-linear iteration step. Therefore, to obtain a static initial condition, it is sufficient to use a rough criterion for the fixed-point iteration procedure. In each such iteration, n_C+1 linear equation systems are to be solved in every time-step.

IV. NUMERICAL INVESTIGATIONS

The methods described will be validated on a single-phase and a three-phase transformer problem to point out the advantages and disadvantages. A comparison with an approach without an initialization will also be presented.

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